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RELATIONSHIP BETWEEN TURBULENT AND KINETIC ENERGY IN A MIXED SUBSTANCE

V. E. Neuvazhaev

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A simple semi-empirical model of turbulent mixing is used to calculate the eddy kinetic energy of a mixed zone and to compare this value with the kinetic energy of the zone. The latter is determined by the effect of acceleration, which induces motion in the corresponding substance. This problem was examined in [1] on the basis of the model in [2]. Below, we use the approximate approach developed in [3, 4]. We compare our results with the results obtained in [1] and explain the differences – particularly for the case of impulsive accumulation. Data for impulsive acceleration is compared with experimental results in [5] and satisfactory agreement is established. This agreement could be improved if the method used to analyze the empirical data is chosen so as to be consistent with the theoretical method.

Formulation of the Problem. We will examine the problem of the mixing of two incompressible fluids of different densities located in a gravitational field. The direction of the field is such as to induce an instability which leads to turbulent mixing of the substances. Mikaelian [1] used the diffusion model in [2] to calculate the relation between the change in potential energy due to turbulent mixing and the kinetic energy acquired by a mixed substance as a result of acceleration.

We calculate the change in potential energy which is due to mixing of the substance within the interval $x_2 \leq x \leq x_1$:

$$\Delta\Pi = g_0\left[\int_{x_2}^{0}(\rho_2-\rho)\,x\,dx+\int_{0}^{x_1}(\rho_1-\rho)\,x\,dx\right]. \qquad (1)$$

Here g_0 is acceleration; ρ is the density of the mixture; ρ_2 and ρ_1 are the densities of the light and heavy fluids; $x = 0$ is the position of the interface at the initial moment when the fluids are not yet mixed.

We used a notation different than that employed in [1] for the change in potential energy, in that E_t is taken to mean the turbulence energy due to the characteristic turbulent velocity v. Following [1] in designating the kinetic energy of the mixed substance as E_d, we can determine this quantity as

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$$E_d = (1/2)\left(\int_0^t g\,dt\right)^2 \int_{x_2}^{x_1} \rho\,dx. \tag{2}$$

The ratio $\Delta\Pi/E_d$ was calculated in [1] with the assumption that the profile of density ρ depends linearly on x. This value was then refined with allowance for the specific density profiles corresponding to the chosen diffusion model – which coincided with the model in [2].

A special case is impulsive acceleration, which corresponds roughly to the passage of a shock wave through the interface. Mikaelian [1] used a model which leads to a linear in crease in the size of the mixing zone over time. However, this is valid only near the beginning of acceleration. As was shown in [3, 4], the width of the mixing zone will increase over time in accordance with the law $t^{2/7}$. This finding was made on the assumption that some of the turbulence energy is converted to heat. Allowing for this fact, below we calculate the ratio $\Delta\Pi/E_d$ for both constant and impulsive acceleration. We based our calculations on the ℓv-model developed in [6]. Here, we made use of the approximate approach in [7], which leads to simple analytical relations.

The main idea underlying the study [7] is that characteristic turbulent velocity v in the mixing region is assumed to be independent of the space variable. After averaging over the mixing region, the balance equation that we constructed for v takes the form

$$d\bar{v}^2/2d\tau + k\bar{v}^2/\tau = gA/(2\eta_1\sqrt{\tau}),\ d\tau = \alpha L\bar{v}dt; \tag{3}$$

$$f_1 = (\rho - \rho_2)/(\rho_1 - \rho_2) = 0{,}5\,(1 + \Phi(\eta)),\ L = 4\eta_1\sqrt{\tau}, \tag{4}$$

$$\eta = x/2\tau^{0{,}5},\ k = 0{,}25 + \nu/16\eta_1^2\alpha^2 + A^2/24,\ A = (\rho_1 - \rho_2)/(\rho_1+\rho_2),\ \eta_1 = 2/\sqrt{\pi},$$

$$\Phi(\eta) = (2/\sqrt{\pi})\int_0^\eta \exp(-\eta^2)\,d\eta$$

(α and ν are constants chosen on the basis of comparison with experimental results: $\alpha = 0.287$, $\nu = 16\eta_1^2\alpha^2$). This choice ensures agreement with the results in [8, 9] for a constant acceleration $L = 0.14\ Ag_0t^2$ and a "2/7" law. Strictly speaking, the constant 0.14 should have been recalculated, with the width of the mixing region in the experiment being determined not over the front but integrally by means of the formula

$$L = 2\left(\int_{x_2}^{0} (f_1/f_1(0))\,dx + \int_0^{x_1} (f_2/f_2(0))\,dx\right). \tag{5}$$

However, we did not do this because it would have led to only a slight reduction in the constant 0.14.

The right side of Eq. (3) is the source of turbulent mixing. In the case of an impulsive acceleration law, it is always taken with a + sign. The second term in the left side characterizes the dissipation of turbulence energy. More precisely, the conversion of turbulence energy into heat is determined by the second and third terms in the coefficient k. If these terms are excluded, then the potential energy will be equal to the kinetic energy associated with turbulent mixing.

Constant Acceleration. In the model being used here, the volume concentration f_1 – characterizing the distribution of density – is independent of the Atwood number A. Figure 1 compares it with the concentration obtained in [1] [line 1 – Eq. (4), lines 2 and 3 – from [1] with A = 0.1 and 1, line 4 – Eq. (5)]. The solution for eddy kinetic energy and the width of the mixing zone has the form

$$\bar{v}^2 = g_0AL/(2\eta_1^2(1 + 4k)) = 0{,}065g_0AL/(1 + 0{,}028A^2),$$
$$L = 8\eta_1^2\alpha^2g_0At^2/(1 + 4k) = 0{,}14g_0At^2/(1 + 0{,}028A^2). \tag{6}$$

The change in potential energy $\Delta\Pi$ and kinetic energy E_d are calculated from Eqs. (1) and (2) with the use of the expression for density (4). We finally obtain

$$\Delta\Pi/E_d = LA/4\eta^2g_0t^2 = 0{,}0275A^2/(1 + 0{,}028\ A^2), \tag{7}$$

where the change in potential energy $\Delta\Pi$ is found from Eq. (1) with the assumption that $-\infty < x < \infty$. With allowance for (6), we determine the fraction of eddy kinetic energy as

$$E_t/E_d = \bar{v}^2/g_0^2t^2 = 0{,}0092A^2/(1 + 0{,}028A^2)^2. \tag{8}$$

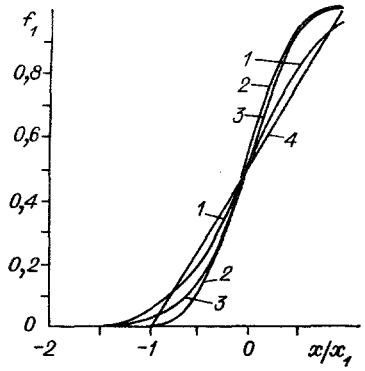

Fig. 1

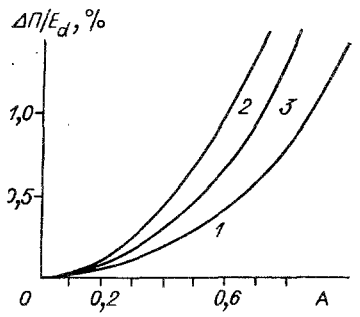

Fig. 2

Relations (7) and (8) are shown in Fig. 2 (line 1 is from [1], line 2 is from Eq. (7)) in comparison with the results from [1]. There is a significant difference between the energy ratios in (7) (with A = 1, (7) = 0.0267, while in [1] (7) = 0.014), this difference being due mainly to different profiles of volume concentration. In making the comparison, we combined the mixing fronts (see Fig. 1). If Mikaelian [1] had done as we did and replaced the profile by a straight line in order to conserve the mass of the mixed substance, then his results (line 3 in Fig. 2) would have been appreciably closer to ours. It is better to make such comparisons with experiments in which the density profile is measured [8, 9] and the approximation of the initial data is less regular. The ratio $\Delta\Pi/E_d$ from [1] was found with allowance for the fact that the mixing width in [1] and here are related by the formula $L = (5/8)L_3$, where L is the width determined from Eq. (5). With the use of the model in [2], both this formula and the analytical expression for $\Delta\Pi/E_d$ are obtained in the appendix for the case of small A.

Impulsive Acceleration. Equation (3) can have a nontrivial solution only in the presence of initial roughness ($L_0 \neq 0$) [10]. In this case, the solution of the equations in the previous section takes the form

$$\bar{v}^2 = \bar{v}_1^2(\beta)(L_1/L)^{4k}; \tag{9}$$

$$L = L_1\left[1 + 8\eta_1^2\alpha(1+k)\bar{v}_1(\beta)(t-t_0)/L_1\right]^{1/(1+2k)}. \tag{10}$$

Here, $\bar{v}_1(\beta)$ is the eddy velocity imparted by the shock wave; β is a dimensionless parameter determined below; t_0 is the time of passage of the shock wave across the boundary, eroded to the width L_0. Here, we write E_t and E_d as

$$E_t = 0{,}25\bar{v}_1^2(\beta)(\rho_1+\rho_2)L_1^{4k}L^{1-4k},\quad E_d = 0{,}25U_0^2(\rho_1+\rho_2)L$$

($U_0 = \int_0^{t_0} g\,dt$ is the velocity acquired by the boundary due to impulsive acceleration). Their ratio is

$$E_t/E_d = \left(\bar{v}_1^2(\beta)/U_0^2\right)(L_1/L)^{4k}. \tag{11}$$

It should be noted that, at k = 0.25, the eddy kinetic energy transmitted by the shock wave will remain constant. In the case of the "2/7" law, 4k = 5, and energy dissipation takes place with the exponent 5. The value of $\bar{v}_1^2(\beta)/U_0^2$ is found within the interval $[4\alpha^2A^2/(1+4k)^2,\ 4\alpha^2A^2]$. We represent the solution in the form

$$\bar{v}_1^2(\beta)/U_0^2 = (0{,}065AL_1/(\beta L_0))\left[1-(L_0/L_1)^6\right],$$
$$L_1/L_0 = \begin{cases} 1 + 10{,}2\alpha^2\beta A, & \text{if } \beta A \leqslant 0{,}24, \\ (0{,}924 + 1{,}23\alpha\sqrt{\beta A})^2, & \text{if } \beta A \geqslant 0{,}24, \end{cases} \tag{12}$$

where $\beta = U_0t_0/L_0$. The leftmost value in the interval indicated above is realized at large β.

Let us examine Eq. (11) with $L = L_1$: $E_t/E_d = \bar{v}_1^2(\beta)/U_0^2$. This relation has a quadratic dependence on A, while its coefficient can vary $(1+4k)^2 = 36$ times at the extreme values of the parameter β:

$$E_t/E_d = (0{,}01 - 0{,}37)\,A^2. \tag{13}$$

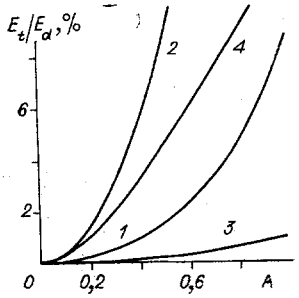

Fig. 3

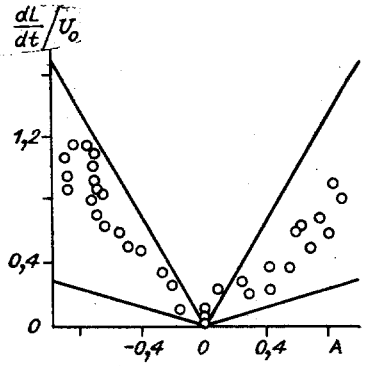

Fig. 4

Other values of β correspond to a more complex law determined in accordance with Eq. (11). For comparison, the case β = 1 is shown in Fig. 3 with $L = L_1$ (line 1 shows results from [1]; lines 2-4 correspond to β = 0, ∞, and 1).

Impulsive Acceleration. Comparison with Experiments in [5, 11]. Zaitsev et al. [5] measured the dependence of the dimensionless quantity $(dL/dt)/U_0$ on A and established that a linear relation of the following form holds when the first shock wave crosses the interface

$$(dL_1/dt)/U_0 = 0{,}02 + 0{,}068(0{,}070)|A| \tag{14}$$

After the second wave – when the eroded region of the boundary is 10 mm wide – the coefficient with the Atwood number increases by approximately one order:

$$(dL_2/dt)/U_0 = 0{,}05 + 1{,}15(0{,}85)|A|. \tag{15}$$

The number in the parentheses shows the value of the coefficient with the motion of the shock wave from the heavier medium to the lighter medium.

Let us compare these results with the theoretical conclusions in the previous section. To do this, we will follow the approach we used in deriving Eqs. (14) and (15) and examine Eq. (11) with $L = L_1$ [having inserted this equation into (12)]. In order to find the relationship between the (14) and (15), we take the square root of both sides of (10). We then have

$$\sqrt{(E_t/E_d)(L = L_1)} = \bar{v}_1(\beta)/U_0 = (dL/dt)/(U_0 8\eta_1^2\alpha). \tag{16}$$

Here, we used the equation $dL/dt = 8\eta_1^2\alpha\bar{v}_1$, which follows from the formulas in the first section. Thus, it follows from a comparison of (12) and (16) that $(dL/dt)/U_0 = 8\eta_1^2\alpha\bar{v}_1(\beta)$.

Figure 4 compares theoretical relations (lines) with experimental results [5] (circles). The latter are within the limits established by the theory, the only exception being the first shock wave. The film turned out to have a substantial effect when this wave passed.

Appendix. Analytical Representation of the Solution with Small Values of A. The profile of volume concentration has the form [2]

$$f_1 = 0{,}5 + 15x/8L_3 - 5(x/L_3)^3 + 6(x/L_3)^5, \tag{17}$$

where it is assumed that mixing is symmetric at small A. Thus, $-L_3/2 \le x \le L_3/2$. Calculation of the ratio $\Delta\Pi/E_d$ leads to the expression

$$\Delta\Pi/E_d = L_3 A/(14 g_0 t^2). \tag{18}$$

We use Eq. (5) to change over to the width L determined from the mass conservation law for the mixture. By substituting Eqs. (17) and (18) $L = (5/8)L_3$ is obtained. With allowance for the latter equation, Eq. (18) takes the form $\Delta\Pi/E_d = 0.016\ A^2$. This formula is valid for small values of A.

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TURBULENT FLOW AND HEAT TRANSFER OF A CHEMICALLY REACTING GAS MIXTURE IN A CHANNEL BEHIND AN ACCELERATING PISTON

A. M. Bubenchikov and S. N. Kharlamov

UDC 536.24

This article examines the nonsteady turbulent motion of a recombining gas in a chamber behind an accelerating piston. The chamber is a section of a cylindrical tube bounded on the left by a stationary wall and on the right by the piston. The evacuated section of the channel is located to the right of the piston. Before beginning motion, the partly dissociated gas – at a fairly high pressure – is uniformly distributed over the entire volume of the chamber, while the position of the piston is fixed. The piston is released at a certain moment of time taken as the initial moment and begins to accelerate toward the free end of the tube under the pressure of the hot gas. Expansion of the region occupied by the gas and the exchange of heat with the relatively cold wall of the channel lead to intensive recombination in the flow.

Our goal here is to construct a mathematical model of the given process and to study its gasdynamic features and criterional relations to determine parameters of the dynamic and thermal effects of the flow on the channel wall.

To describe the gas flow in the present case, it is best to use the Reynolds equations in the "narrow channel" approximation [1]. Together with the energy equations for a two-component, chemically active mixture and the heat-conduction equation for the wall, these equations have the form

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(\rho v r)=0; \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{\Sigma}\frac{\partial u}{\partial r}\right),\quad \frac{\partial p}{\partial r}=0; \tag{2}$$

$$\rho\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial r}\right)=\frac{dp}{dt}+\mu_{\Sigma}\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho a_{\Sigma}\frac{\partial h}{\partial r}+r\rho a\,(\mathrm{Le}-1)\,\Delta h\frac{\partial c}{\partial r}\right],\quad p=\rho\tilde{R}T\,(1+c)/(2M); \tag{3}$$

$$\rho_w c_w\frac{\partial T_w}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_w\frac{\partial T_w}{\partial r}\right),\quad \frac{dp}{dt}=\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x},\quad \mu_{\Sigma}=\mu+\mu_t,\quad a_{\Sigma}=a+a_t, \tag{4}$$

where t is time; x and r are cylindrical coordinates; u and v are components of the velocity vector; ρ, h, and p are the density, enthalpy, and pressure; μ and a are molecular viscosity and diffusivity; μ_t and a_t are the turbulence analogs of the transport coefficients; M and c

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